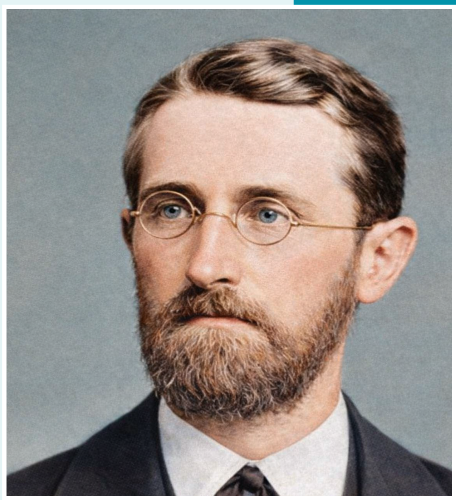
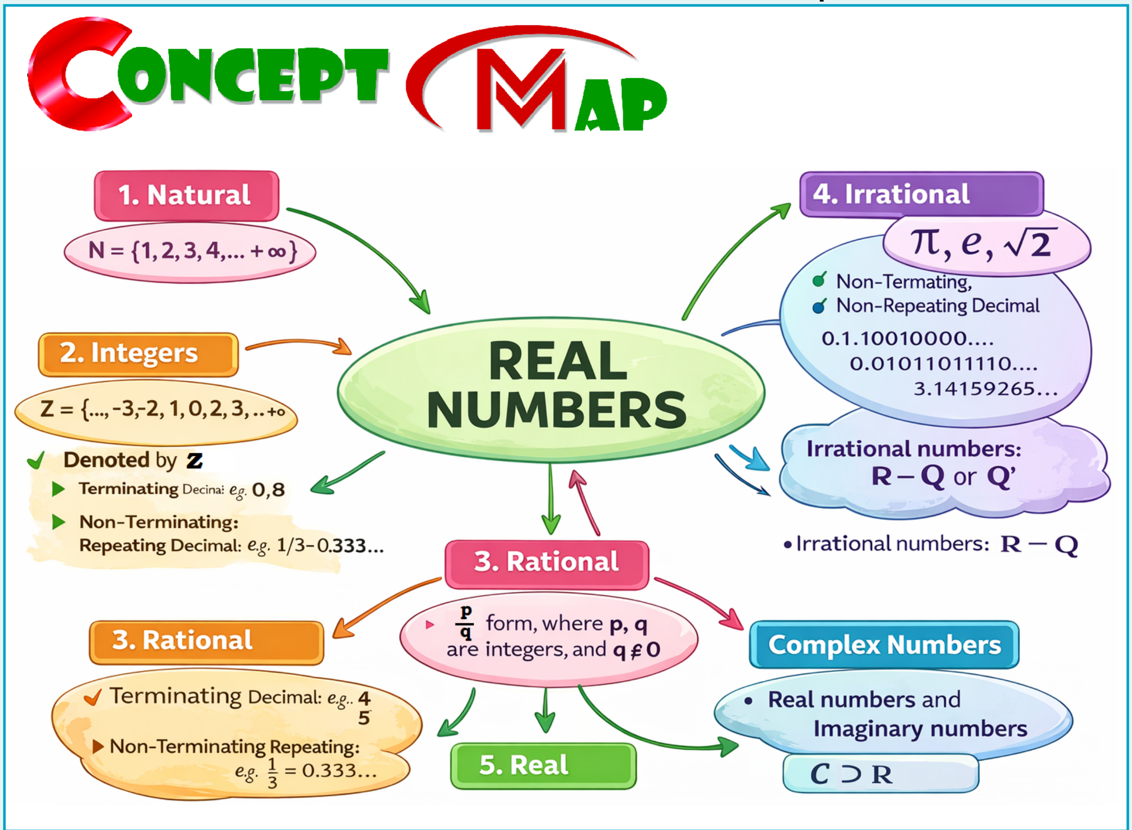
1 REAL NUMBERS

Julius Wilhelm Richard Dedekind, was a German mathematician who developed a major redefinition of irrational numbers in terms of arithmetic concepts. Although not fully recognized in his lifetime, his treatment of the ideas of the infinite and of what constitutes a real number continues to influence modern mathematics. Dedekind developed the idea that both rational and irrational numbers could form a continuum (with no gaps) of real numbers, provided that the real numbers have a one-to-one relationship with points on a line.





Concept 1

Real Numbers:

The collection of all rational numbers and irrational numbers together is the set of real numbers. It is represented by R.

$$R = Q \cup Q'$$

Rational Numbers:

Numbers that can be written in the form $\frac{p}{q}$, where p and q are integers having no common factors other than 1 and $q \neq 0$, are known as rational numbers.

Rational numbers denoted by Q.

Note: Every integer is a rational number, since in an integer denominator can be taken as 1.

Zero is a rational number but $\frac{1}{0}$ is undefined.

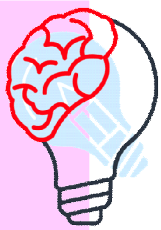
Example: All integers and perfect square numbers.

$\sqrt{1}, \sqrt{4}, \sqrt{9}$ are rationals because $\sqrt{1} = 1, \sqrt{4} = 2, \sqrt{9} = 3$

Terminating and Non-Terminating:

Every rational number can be represented either as a terminating non-repeating decimal or non-terminating repeating (recurring) decimal.

Knowledge Box



- Terminating Decimal: A decimal that stops.
- Non-Terminating Repeating Decimal: A decimal that repeats in a pattern.
- Non-Terminating Non-Repeating Decimal: A decimal that neither stops nor repeats.

Example: $\underbrace{\frac{4}{5} = 0.80, \frac{9}{5} = 1.8, \frac{5}{8} = 0.625}_{\text{Terminating}}$

and

$$\underbrace{\frac{2}{3} = 0.666...., \frac{1}{6} = 0.1666..., \frac{8}{7} = 1.142857142857...}_{\text{Non-terminating but repeating}}$$

If a rational number ($\neq$ integer) can be expressed in the form $\frac{p}{2^n \times 5^m}$, where $p \in Z$, $n \in W$ and $m \in W$ then rational number will be terminating decimal. Otherwise, rational number will be a non–terminating recurring decimal.

Example: (i) $\frac{3}{8} = \frac{3}{2^3 \times 5^0}$ So, $\frac{3}{8}$ is a terminating decimal.

(ii) $\frac{8}{75} = \frac{8}{5^2 \times 3}$ is a non-terminating recurring decimal.

Period and Periodicity:

The recurring part of the non-terminating recurring decimal is called period and the number of digits in the recurring part is called periodicity.

Example: $\frac{1}{3} = 0.\overline{3}$ period = 3, periodicity = 1.

Converting into p/q forms:

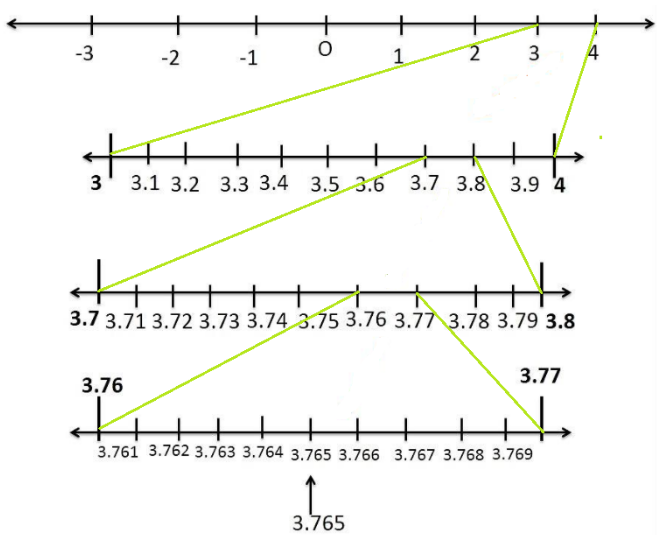
Note: **(i)** $a.b\overline{cd} = \frac{abcd - ab}{10^3 - 10^1}$ **(ii)** $a.\overline{bcd} = \frac{abcd - a}{10^3 - 10^0}$

Example: $0.1\overline{25} = \frac{125 - 1}{10^3 - 10^1} = \frac{124}{990}$; $3.1\overline{25} = \frac{3125 - 31}{10^3 - 10^1} = \frac{3094}{990}$

Successive magnification:

The process of visualisation of representing a decimal expansion on the number line is known as the process of successive magnification.

Visualise the representation of 3.765 on number line.



Surd:

If 'n' is a positive integer and a rational number a (>0) is not the n^{th} power of any other rational number, then $\sqrt[n]{a}$ or $a^{\frac{1}{n}}$ is called a 'surd' or 'radical' of order n and it can be read as nth root of a.

Note: In a surd $\sqrt[n]{a}$,

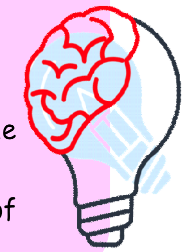
i) The symbol $\sqrt[n]{\ }$ is called radical sign.

ii) 'a' is called the radicand.

iii) 'n' is called the order of the surd.

Knowledge Box

The order of surd is the degree of the root.
For example: $\sqrt[5]{3}$ is a surd of order 5.



Types of surds:

1. **Unit surds and multiples of surds:** If $\sqrt[n]{a}$ is a surd, it is also referred to as a unit surd. If k is a rational number, $k\sqrt[n]{a}$ is a multiple of a surd.

   Note: All multiples of surds can be expressed as unit surds as $k\sqrt[n]{a} = \sqrt[n]{k^n.a}$

2. **Mixed surds:** If a is a rational number (not equal to 0) and $\sqrt[n]{b}$ is a surd, then $a+\sqrt[n]{b}$, $a-\sqrt[n]{b}$ are called mixed surds. If a=0, they are called pure surds.

   Example $2+\sqrt[4]{3}, 5-\sqrt[3]{6}$ are mixed surds, while $\sqrt[4]{3}$, $\sqrt[3]{6}$ are pure surds.

3. **Compound surd:** A surd which is the sum or difference of two or more surds is called a compound surd.

   Example $\sqrt{2}+\sqrt[3]{3}$, $\sqrt{3}+\sqrt[5]{7}-\sqrt[3]{2}$ and $1+\sqrt{2}-\sqrt{3}$ are compound surds.

4. **Binomial surd:** A compound surd consisting of two surds is called a binomial surd.

   Example $\sqrt{3}+3\sqrt{5}$, $\sqrt{6}+4\sqrt{5}$, $\sqrt{8}-\sqrt[3]{7}$

5. **Similar surds:** If two surds are different multiples of the same surd, they are called similar surds. Otherwise they are dissimilar surds.

   Example

   $2\sqrt{2}$, $4\sqrt{2}$, $7\sqrt{2}$ are similar surds. $(\sqrt{2}+3\sqrt{3})$, $2\sqrt{2}+6\sqrt{3}$ are similar surds and $1+\sqrt{2}$, $2+2\sqrt{2}$ are similar surds. $3\sqrt{3}$ and $6\sqrt{5}$ are dissimilar surds.

Self Assessment Test - 01

1. **Which set includes all rational and irrational numbers?**
   (A) Complex numbers
   (B) Imaginary numbers
   (C) Real numbers
   (D) Whole numbers

2. **When does a decimal expansion become terminating?**
   (A) When the remainder never becomes zero
   (B) When the remainder repeats after a certain stage
   (C) When the remainder becomes zero after a certain stage
   (D) When the remainder is always zero

3. **How is the decimal expansion of an irrational number described?**
   (A) Terminating non-recurring
   (B) Non-terminating recurring
   (C) Terminating recurring
   (D) Non-terminating non-recurring

4. **What is the process of representing a decimal expansion on the number line known as?**
   (A) Decimal conversion
   (B) Decimal placement
   (C) Successive magnification
   (D) Decimal visualization

5. **What is the result of dividing one rational number by a non-zero rational number?**
   (A) An irrational number
   (B) A terminating decimal
   (C) A non-terminating recurring decimal
   (D) A rational number

6. **What type of number is the result of adding or subtracting a rational number and an irrational number?**
   (A) Rational number
   (B) Whole number
   (C) Imaginary number
   (D) Irrational number

7. **Which of the following is a compound surd?**
   (A) $\sqrt{3}+2\sqrt{5}$ (B) $4-\sqrt{7}$
   (C) 5 (D) $6-\sqrt{2}$

8. **In the expression $\sqrt[3]{1458}$, what type of surd is it?**
   (A) Unit surd
   (B) Binomial surd
   (C) Compound surd
   (D) Similar surd

9. **What is the order of $\sqrt[6]{27}$?**
   (A) 3 (B) 6
   (C) 9 (D) 27

MARK YOUR ANSWERS WITH PEN ONLY.

1 Ⓐ Ⓑ Ⓒ Ⓓ	2 Ⓐ Ⓑ Ⓒ Ⓓ	3 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

Concept 2

Conjugate Surd:

If an expression is in the form $(a+\sqrt{b})$, its conjugate surd is $(a-\sqrt{b})$ and vice versa.

Property of Conjugate Surds:

When you multiply a surd by its conjugate, the result is a rational number (a number without any surds).

$$(a+\sqrt{b})\times(a-\sqrt{b})=a^2-b$$

Example: Conjugate of $(3+\sqrt{5})$ is $(3-\sqrt{5})$.

Multiplying

$$(3+\sqrt{5})\times(3-\sqrt{5})=9-5=4$$

Misconception :

Misconception: Some believe rationalizing always eliminates the surd. It only simplifies expressions.

Correction: The surd remains in same form.

Rationalization:

If the product of two surds is a rational number, then each of them is called a rationalizing factor (R.F) of the other.

Example: $\sqrt[3]{32}\times\sqrt[3]{2}=\sqrt[3]{64}=\sqrt[3]{4^3}=4$ is a rational number.

$\therefore$ The R.F. of $\sqrt[3]{32}$ is $\sqrt[3]{2}$ and R.F. of $\sqrt[3]{2}$ is $\sqrt[3]{32}$

Note:

(i) The R.F. of a given surd is not unique. A surd has infinite number of rationalizing factors.

(ii) $\sqrt[n]{a^{n-m}}$ is the rationalizing factor of $\sqrt[n]{a^m}$ and vice-versa.

(iii) When an expression is multiplied and divided by the same number (rational or irrational); the value of the expression remains unchanged.

Laws of radicals:

If a>0, b>0 and n is a positive rational number, then

1. $\left(\sqrt[n]{a}\right)\left(\sqrt[n]{b}\right)=\sqrt[n]{ab}$

2. $\sqrt[m]{\sqrt[n]{a}}=\sqrt[mn]{a}=\sqrt[n]{\sqrt[m]{a}}$

3. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\dfrac{a}{b}}$

4. $\sqrt[n]{a^p}=a^{p/n}$ and $\sqrt[n]{a^p}=\sqrt[n]{\sqrt[m]{(a^p)^m}}$

Rationalization of Denominator:

Rationalising factor for $\frac{1}{\sqrt{a}}$ is $\sqrt{a}$, for $\frac{1}{a \pm \sqrt{b}}$ is $a \mp \sqrt{b}$ and for $\frac{1}{\sqrt{a} \pm \sqrt{b}}$ is $\sqrt{a} \mp \sqrt{b}$.

Example:

Suppose that $x = \frac{11}{4-\sqrt{5}}$. Find the value of $x^2-8x+11$

Solution:

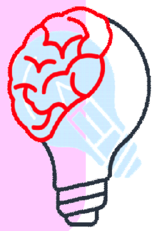
We rationalize the denominator of x:

$$x = \frac{11}{4-\sqrt{5}} \times \frac{4+\sqrt{5}}{4+\sqrt{5}} = \frac{11(4+\sqrt{5})}{16-5} = 4+\sqrt{5}$$

$$\Rightarrow x-4 = \sqrt{5}$$

Knowledge Box

$\sqrt[n]{a^{n-m}}$ is the rationalizing factor of $\sqrt[n]{a^m}$ and vice-versa.



Now, we square both the sides of this relation we have obtained:

$(x-4)^2 = 5 \Rightarrow x^2-8x+16=5 \Rightarrow x^2-8x+11 = 0$

Example:

Suppose that a and b are rational numbers such that $\frac{3+2\sqrt{3}}{5-2\sqrt{3}} = a+b\sqrt{3}$. Find the value of a and b.

Solution:

We rationalize the denominator of the left hand side (LHS):

$$\text{LHS} = \frac{3+2\sqrt{3}}{5-2\sqrt{3}} \times \frac{5+2\sqrt{3}}{5+2\sqrt{3}} = \frac{15+6\sqrt{3}+10\sqrt{3}+12}{(5)^2-\left(2\sqrt{3}\right)^2} = \frac{27+16\sqrt{3}}{25-12} = \frac{27}{13}+\frac{16}{13}\sqrt{3}$$

$\therefore$ a = 27/13 and b = 16/13.

Inspection Method:

In finding the square root of $9+4\sqrt{5}$, we first write it as $9+2\sqrt{20}$ and the two numbers, namely 4 and 5 are found out whose sum is 9 and product is 20.

Thus, $9+4\sqrt{5} = 9+2\sqrt{20} = 4+5+2\sqrt{4}\sqrt{5} = \left(\sqrt{4}+\sqrt{5}\right)^2 = \left(2+\sqrt{5}\right)^2$

$$\therefore \sqrt{9+4\sqrt{5}} = \pm\left(2+\sqrt{5}\right)$$

Self Assessment Test - 02

1. What is the conjugate of the surd $4+\sqrt{5}$?

(A) $4-\sqrt{5}$ (B) $-4+\sqrt{5}$

(C) $\sqrt{4}-5$ (D) $\sqrt{4}+5$

2. What is the result of multiplying the surd $3+\sqrt{7}$ by its conjugate?

(A) 9+7 (B) 9−7

(C) $9+\sqrt{49}$ (D) 9−49

3. Which of the following is the correct rationalization of $\frac{1}{2+\sqrt{3}}$?

(A) $\frac{2-\sqrt{3}}{1}$ (B) $\frac{2-\sqrt{3}}{4+3}$

(C) $\frac{2-\sqrt{3}}{4+\sqrt{3}}$ (D) $\frac{2+\sqrt{3}}{4-3}$

4. How would you rationalize the expression $\frac{1}{\sqrt{2}+\sqrt{5}}$

(A) Multiply and divide by $\sqrt{2}-\sqrt{5}$

(B) Multiply and divide by $\sqrt{2}+\sqrt{5}$

(C) Multiply and divide by $2+\sqrt{5}$

(D) Multiply and divide by $\sqrt{2}$

5. What is the result of rationalizing the denominator of $\frac{1}{\sqrt{3}-1}$?

(A) $\frac{1}{2}$ (B) $\frac{3+1}{2}$

(C) $\frac{\sqrt{3}-1}{2}$ (D) $\frac{\sqrt{3}+1}{2}$

6. How would you rationalize the denominator of $\frac{3}{5+\sqrt{2}}$?

(A) Multiply by $\frac{5-\sqrt{2}}{5-\sqrt{2}}$

(B) Multiply by $\frac{5+\sqrt{2}}{5+\sqrt{2}}$

(C) Multiply by $\frac{\sqrt{2}-5}{\sqrt{2}-5}$

(D) Multiply by $\frac{5}{5}$

7. Simplify $\sqrt{12}-\sqrt{32}-\sqrt{48}$. What is the resulting surd?

(A) $8\sqrt{3}-24\sqrt{2}-12\sqrt{3}$

(B) $8\sqrt{3}-24\sqrt{2}-8\sqrt{3}$

(C) $-4\sqrt{2}-2\sqrt{3}$

(D) $-4\sqrt{3}-24\sqrt{2}+12\sqrt{3}$

MARK YOUR ANSWERS WITH PEN ONLY.

1 Ⓐ Ⓑ Ⓒ Ⓓ	2 Ⓐ Ⓑ Ⓒ Ⓓ	3 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

Concept 3

Irrational Numbers:

A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an irrational number.

Thus, non-terminating, non-repeating decimals are irrational numbers. These are denoted by Q|.

Examples of Irrational Numbers:

Type 1: i) Clearly, 0.01001000100001... is a non-terminating and non-repeating decimal and therefore, it is irrational,

ii) 0.12112111211112..., 0.54554555455554... are irrationals.

Type 2: $\sqrt{2}$ =1.4142135623731...is an irrational number. Later Theodoras of Cyrene showed that $\sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{10}, \sqrt{11}, \sqrt{12}, \sqrt{13}, \sqrt{14}, \sqrt{15}$ and $\sqrt{17}$ are also irrational numbers.

Type 3: The number 'e' (Euler's number) is an irrational. Its value is 2.71828182845...i.e., 2 < e <3

π = 3.14159265358979323 8 . . . It is an irrational number.

Properties of Irrational Numbers:

1. Irrational numbers satisfy the commutative, associative and distributive laws for addition and multiplication.

2. i) Sum of two irrationals need not be an irrational.

Ex. Each one of $\left(2+\sqrt{3}\right)$ and $\left(4-\sqrt{3}\right)$ is irrational.

But, $\left(2+\sqrt{3}\right)+\left(4-\sqrt{3}\right)=6$, which is rational,

ii) Difference of two irrationals need not be an irrational.

Ex. Each one of $\left(5+\sqrt{2}\right)$ and $\left(3+\sqrt{2}\right)$is irrational.

But $\left(5+\sqrt{2}\right)-\left(3+\sqrt{2}\right)=2$, which is rational,

iii) Product of two irrationals need not be an irrational.

Ex. $\sqrt{3}$ is irrational. But $\sqrt{3}\times\sqrt{3}=3$, which is rational,

iv) Quotient of two irrationals need not be an irrational.

Ex. Each one of $2\sqrt{3}$ and $\sqrt{3}$ is irrational. But $\frac{2\sqrt{3}}{\sqrt{3}}=2$, which is rational.

Ex: Prove that $\sqrt{2}+\sqrt{3}$ is an irrational.

Sol: Let us suppose that $\sqrt{2}+\sqrt{3}$ is rational.

Let $\sqrt{2}+\sqrt{3}=\frac{a}{b}$, where a, b are integers and $b \neq 0$

Therefore, $\sqrt{2}=\frac{a}{b}-\sqrt{3}$

Squaring on both sides, we get

$2=\frac{a^2}{b^2}+3-2\frac{a}{b}\sqrt{3}$

Rearranging $\frac{2a}{b}\sqrt{3}=\frac{a^2}{b^2}+3-2$

$=\frac{a^2}{b^2}+1$

$\sqrt{3}=\frac{a^2+b^2}{2ab}$; Irrational $\neq$ rational

Therefore $\sqrt{2}+\sqrt{3}$ is irrational.

Ex: Prove that $\sqrt{2}$ is irrational.

Sol. Let us assume $\sqrt{2}$ is rational.

$\frac{p}{q}(q \neq 0)$, where p, q are co-prime numbers

$\sqrt{2}=\frac{p}{q}$ (Squaring both side)

$\left(\sqrt{2}\right)^2=\frac{p^2}{q^2}$

$2q^2=p^2 \Rightarrow q^2=\frac{p^2}{2} \Rightarrow$ 2 divides p^2 also divides p.

Let p = 2k

$q^2=\frac{4k^2}{2} \Rightarrow k^2=\frac{q^2}{2} \Rightarrow$ 2 divides q^2 also divides q.

Therefore 2 is common factor for p and q.
This contradicts our assumption.

So, $\sqrt{2}$ is irrational.

Self Assessment Test - 03

1. Numbers which cannot be expressed in the form $\frac{p}{q}, q \neq 0, p, q \in Z$ are called ____.

(A) rational

(B) irrational

(C) fractional

(D) decimal

2. The number of irrationals in the given list $\sqrt{3}, \pi, \frac{1}{3}, 0, \sqrt[5]{2}, \frac{22}{7}, \sqrt{36}$ is

(A) 3

(B) 4

(C) 5

(D) 6

3. The sum of two Irrational numbers is

(A) Either Rational or Irrational number

(B) Always Rational numbers

(C) Always Irrational numbers

(D) All of the above

4. $\sqrt{2}$ is an ____ number.

(A) rational

(B) irrational

(C) natural

(D) fractional

5. Identify the correct property of Irrational numbers?

(A) Product of two Irrationals is Irrational

(B) Division of one Rational and one Irrational is either Rational or Irrational

(C) Quotient of one Rational and are Irrational is either Rational or Irrational

(D) Difference of a Rational and an Irrational is always Irrational

6. Irrational among the following is

(A) $\sqrt{2} \times \sqrt{2}$

(B) $\sqrt{21} \times \sqrt{\frac{3}{7}}$

(C) $\sqrt{2 \times 3 \times 5 \times 7}$

(D) $\sqrt{2 \times \frac{2008}{1004} \times 1}$

MARK YOUR ANSWERS WITH PEN ONLY.

1 Ⓐ Ⓑ Ⓒ Ⓓ	2 Ⓐ Ⓑ Ⓒ Ⓓ	3 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

Concept 4

Introduction to Logarithms:

If N and $a(\neq 1)$ are any two positive real numbers and for some real x, such that $a^x = N$, then x is said to be logarithm of N to the base 'a'. It is written as $\log_a N = x$.

Thus, $\boxed{a^x = N \Leftrightarrow \log_a N = x}$

It should be noted that "log" is abbreviation of the word "logarithm".

Logarithms are defined only for positive real numbers.

There exists a unique 'x' which satisfies the equation $a^x = N$.

So $\log_a N$ is also unique.

Ex:

Exponential form	Logarithmic form
$27^x = 1$	$\log_{27} 1 = x$
$5^x = 125$	$\log_5 125 = x$
$10^x = 1000$	$\log_{10} 1000 = x$
$10^x = \frac{1}{100}$	$\log_{10} \frac{1}{100} = x$
$27^x = \frac{1}{3}$	$\log_{27} \frac{1}{3} = x$
$\frac{1}{3}^x = 27$	$\log_{\frac{1}{3}} 27 = x$

Types of Logarithms:

There are two systems of logarithms which are generally used.

Natural Logarithms:

In this system, the base of the logarithm is taken as 'e', where e is an irrational number lying between 2 and 3. The approximate value of e correct to four decimal places is 2.7183 and e is defined as an irrational number given by

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots\ldots = 2.718281828459145\ldots.$$

Logarithms to the base =2.718... are called Natural logarithms or Napierian logarithms.

We often denote $\log_e x$ as lnx.

Common Logarithms:

In this system, the base is always taken as 10. This system is also known as Briggs' system. In all practical calculations this system is every helpful.

If no base is mentioned, the base is always taken as 10.

$\log 10 = \log_{10} 10 = 1$, $\log 100 = \log_{10} 100 = 2$, $10g1000 = \log_{10} 1000 = 3$ etc

Note: $\log_e x = 2.303 \log_{10} x$, when $x > 0$.

Compound Logarithms:

Which of those logarithms its bases are other than '10' and 'e' then those logarithms are called compound logarithms.

Fundamental Laws of Logarithms:

FIRST LAW: If m, n are positive rational numbers, then

$\log_a(mn) = \log_a m + \log_a n$

SECOND LAW: If m and n are positive rational numbers, then

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

THIRD LAW: If m, n are positive rational numbers, then $\log_a (m^n) = n.\log_a m$

FOURTH LAW: $\log_a 1 = 0$. i.e., the log of 1 to any base is always zero

FIFTH LAW: $\log_a a = 1$. i.e., the log of any positive quantity of the same base is always one.

SIXTH LAW: If 'a' is a positive real number and m, n is a positive rational number, then $\log_a n^m = m \times \log_a n$

SEVENTH LAW: If 'a' is a positive real number and 'n' is a positive rational number, then $\log_{a^q} n^p = \frac{p}{q}\log_a n$

EIGHTH LAW: If m is positive rational number a and b are positive real numbers such that $a \neq 1, b \neq 1$, then $\log_a m = \frac{\log_b m}{\log_b a}$

NINTH LAW: If 'a' is positive real number and n is a positive rational number, then $a^{\log_a n} = n$.

Self Assessment Test - 04

1. The logarithms which are calculated to the base '10' are called:

(A) Common logarithms

(B) natural logarithms

(C) Compound logarithms

(D) None

2. The logarithms whose bases other than 10 and e are called

(A) Common logarithms

(B) natural logarithms

(C) Compound logarithms

(D) None

3. The value of log1 to any non-zero base is always equal to ____.

(A) 0

(B) 1

(C) 3

(D) 4

4. The value of logarithm of a number to the same number(other than one) as a base is always equal to ____.

(A) 2

(B) 3

(C) 4

(D) 1

5. If 'a' is positive real number and n is a positive rational number, then $a^{\log_a n}$

(A) 0

(B) 1

(C) n

(D) a

6. $\log_{2\sqrt{2}} 512 =$

(A) 4

(B) 5

(C) 6

(D) 7

7. $7^0 = 1 \Leftrightarrow \log_7 1 = ?$

(A) 1

(B) 0

(C) 2

(D) 3

8. $\log_3 11 . \log_{11} 13 . \log_{13} 15 . \log_{15} 27 =$

(A) 1

(B) 2

(C) 3

(D) None of these

MARK YOUR ANSWERS WITH PEN ONLY.

1 Ⓐ Ⓑ Ⓒ Ⓓ	2 Ⓐ Ⓑ Ⓒ Ⓓ	3 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

C.D.F. (Concepts, Definitions and Formulae)

1. **Unit surds and multiples of surds:** If $\sqrt[n]{a}$ is a surd, it is also referred to as a unit surd. If k is a rational number, $k\sqrt[n]{a}$ is a multiple of a surd.

2. **Mixed surds:** If 'a' is a rational number (not equal to 0) and $\sqrt[n]{b}$ is a surd, then $a+\sqrt[n]{b}$, $a-\sqrt[n]{b}$ are called mixed surds. If a=0, they are called pure surds.

3. **Compound surd:** A surd which is the sum or difference of two or more surds is called a compound surd.

4. **Binomial surd:** A compound surd consisting of two surds is called a binomial surd.

5. **Similar surds:** If two surds are different multiples of the same surd, they are called similar surds. Otherwise they are dissimilar surds.

6. Two mixed surds $a+c\sqrt{b}$ and $d+e\sqrt{b}$ are equal if and only if their respective rational parts and the irrational parts are equal, i.e., a=d and c=e.

7. **LAWS OF RADICALS:**

   If a>0, b>0 and n is a positive rational number, then

   1. $\left(\sqrt[n]{a}\right)\left(\sqrt[n]{b}\right)=\sqrt[n]{ab}$
   2. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\dfrac{a}{b}}$
   3. $\sqrt[m]{\sqrt[n]{a}}=\sqrt[mn]{a}=\sqrt[n]{\sqrt[m]{a}}$
   4. $\sqrt[n]{a^p}=a^{p/n}$ and $\sqrt[n]{a^p}=\sqrt[n]{\sqrt[m]{(a^p)^m}}$

8. **Logarithmic Laws:**

   $\log_a(mn) = \log_a m + \log_a n$

   $\log_a\left(\dfrac{m}{n}\right)=\log_a m-\log_a n$

   $\log_a (m^n) = n.\log_a m$

   $\log_a 1 = 0$

   $\log_a a = 1$

   $\log_a n^m = m\times\log_a n$

   $\log_{a^a} n^p=\dfrac{p}{q}\log_a n$; $\log_a m=\dfrac{\log_b m}{\log_b a}$; $a^{\log_a n}=n$

9. **ORDER OF A SURD:**

   In the surd $\sqrt[n]{a}$, n is called the order of the surd. Thus the orders of $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{5}$ are 2, 3 and 4 respectively.

Advanced Worksheet

LEVEL 1

Single Correct Answer Type (S.C.A.T.):

1. Let x and y be rational an irrational numbers respectively, then x+y is:

(A) A whole number

(B) A rational number

(C) An irrational number

(D) A natural number.

2. If a and b are two positive rational number then $\sqrt{\frac{a+b}{2}}$ is ____ number.

(A) Rational

(B) Irrational

(C) Integer

(D) Either rational or irrational

3. Decimal representation of a rational number cannot be

(A) Terminating

(B) Non-terminating

(C) Non-terminating repeating

(D) Non-terminating non-repeating

4. Decimal representation of an irrational number is always.

(A) Terminating

(B) Terminating repeating

(C) Non-terminating, repeating

(D) Non-terminating, non-repeating

5. If $x = \frac{1}{(2+\sqrt{3})}$, find the value of $x^3 - x^2 - 11x + 3$

(A) 0 (B) 3

(C) 3x (D) 4

6. If $x = (3+\sqrt{8})$, find the value of $\left(x^2 + \frac{1}{x^2}\right)$.

(A) 35+2 (B) 34+1

(C) 6^2 (D) (30+4)

7. $0.2\overline{45}$ as a fraction in simplest form

(A) $\frac{27}{110}$ (B) $\frac{243}{900}$

(C) $\frac{245}{100}$ (D) $\frac{212}{10}$

8. Find x^2, if $x = \frac{\sqrt{\sqrt{5}+2} - \sqrt{\sqrt{5}+2}}{\sqrt{\sqrt{5}+1}}$

(A) 0 (B) 1

(C) 2 (D) 4

9. If $\sqrt{6} = 2.449$, then the value of $\frac{3\sqrt{2}}{2\sqrt{3}}$ is close to:

(A) 1.225 (B) 0.816

(C) 0.613 (D) 2449

10. The number 1.101001000100001... is

(A) a natural number

(B) a whole number

(C) a rational number

(D) an irrational number

11. $\sqrt{7+2\sqrt{10}}$ is equal to

(A) $9\sqrt{10}$

(B) $\sqrt{5}+\sqrt{2}$

(C) $\sqrt{7}-\sqrt{2}$

(D) $8\sqrt{10}$

12. If $x=\sqrt[6]{729}, y=\sqrt[4]{81}, z=\sqrt[5]{32}$ then x+y+z+2 =

(A) 8 (B) 6

(C) 9 (D) 10

13. If two surds are different multiples of the same simple surd, then the surd is

(A) Compound surd

(B) Dissimilar surd

(C) Similar surd

(D) Complex surd

14. $2\sqrt[3]{5}$ in the form of a pure surd is ____.

(A) $\sqrt[3]{40}$

(B) $\sqrt[3]{35}$

(C) $\sqrt[3]{10}$

(D) $\sqrt[3]{32}$

15. If $\sqrt{8}>2.82$ then value of $\frac{2\sqrt{8}}{\sqrt{2}}$ is equal to

(A) 6 (B) 8

(C) 4 (D) 2

16. The value of $\sqrt{7\sqrt{7\sqrt{7\sqrt{7}}}}$ is

(A) 0 (B) 7

(C) $7^{\frac{15}{16}}$ (D) $\sqrt{7}$

17. $\frac{\log_3^{729}+\log_6^{216}}{4+\log_2^{16}-\log_4^{64}}$ = ____.

(A) 9 (B) 4

(C) $\frac{9}{5}$ (D) $\frac{1}{3}$

18. If $\frac{\log 144}{\log 12}=k$ then k =

(A) 0 (B) 3

(C) 1 (D) 2

19. $\log_{1/4}\left(\log_2^3 \log_3^4\right)$ =

(A) 2 (B) –2

(C) $\frac{1}{2}$ (D) $-\left(\frac{1}{2}\right)$

20. For a, b, c are positive rationals other then $\log_{b^3}a^5 . \log_{c^3}b^5 . \log_{a^3}c^5$

(A) $\frac{221}{27}$ (B) $\frac{152}{27}$

(C) $\frac{125}{27}$ (D) $\frac{146}{27}$

LEVEL 2

Multi Correct Answer Type (M.C.A.T.):

21. Which of the following are true

(A) $\left(\sqrt[3]{11}\right)^3 = 11$

(B) $2\sqrt{5} = \sqrt{20}$

(C) $5\sqrt{3}, 5\sqrt{2}$ are dissimilar surds

(D) Order of surd $5\sqrt{4}$ is 4

22. If p = $\log_3 243$ and q = $\log_5 125$, then:

(A) p = 5 (B) q = 3

(C) $p^q = 125$ (D) $q^p = 324$

23. A non-terminating decimal form of the following are:

(A) $\frac{3}{11}$ (B) $\frac{81}{64}$

(C) $\frac{9}{100}$ (D) $\frac{11}{17}$

24. Find any five rational numbers lying between $\frac{2}{7}$ and $\frac{2}{5}$

(A) $\frac{21}{70}, \frac{22}{70}, \frac{23}{70}, \frac{25}{70}, \frac{26}{70}$

(B) $\frac{19}{70}, \frac{20}{70}, \frac{22}{70}, \frac{24}{70}, \frac{26}{70}$

(C) $\frac{18}{70}, \frac{23}{70}, \frac{25}{70}, \frac{27}{70}, \frac{29}{70}$

(D) $\frac{22}{70}, \frac{23}{70}, \frac{24}{70}, \frac{25}{70}, \frac{26}{70}$

25. Which of the following is/are correct?

(A) $\sqrt{5} + \sqrt{3}$ is irrational

(B) $\sqrt{3} - 2$ is rational

(C) $5\sqrt{3}$ is an irrational

(D) π, e are irrational

26. Which of the following is/are correct?

(A) $\sqrt[4]{2} < \sqrt[4]{7}$ (B) $\sqrt[4]{7} < \sqrt[4]{5}$

(C) $\sqrt[5]{10} < \sqrt[5]{13}$ (D) $\sqrt[3]{3} < \sqrt[3]{5}$

27. If $a = \sqrt{2}, b = \sqrt[3]{3}$ and $c = \sqrt[4]{4}$, then which of the following are true?

(A) a > b (B) b > a

(C) b > c (D) a = c

Comprehension Passage Type (C.P.T.):

PASSAGE - I

Two surds can be added or subtracted 1 from the other by using distributive law, only when they are similar surds. We cannot add or subtract dissimilar surds.

28. Simplify: $2\sqrt{12} - 3\sqrt{32} + 2\sqrt{48} =$

(A) $4\sqrt{3} - 12\sqrt{2}$

(B) $12\sqrt{3} - 12\sqrt{2}$

(C) $-4\sqrt{3} - 12\sqrt{2}$

(D) $12\sqrt{3} + 12\sqrt{2}$

29. Simplify: $\sqrt{98} - \sqrt{18} =$

(A) $4\sqrt{3}$ (B) $3\sqrt{4}$

(C) $4\sqrt{2}$ (D) $5\sqrt{3}$

30. Simplify: $\sqrt[3]{48} + \sqrt[3]{162}$ =

(A) $5\sqrt[3]{6}$ (B) $6\sqrt[3]{5}$

(C) $3\sqrt[5]{6}$ (D) $6\sqrt{6}$

PASSAGE - II

If $A = 2\pi\sqrt{\frac{l}{g}}, B = 2\pi, C = \sqrt{\frac{l}{g}}$, then:

31. log A = ____.

(A) log 2

(B) $\log \pi$

(C) $\frac{1}{2}(\log l - \log g)$

(D) $\log 2 + \log \pi + \frac{1}{2}(\log l - \log g)$

32. log B = ____.

(A) log2

(B) $\log \pi$

(C) $\log 2 + \log \pi$

(D) $\log 2 - \log \pi$

33. logC = ____.

(A) log l

(B) log g

(C) $\frac{1}{2}(\log l - \log g)$

(D) $\frac{1}{2}(\log l + \log g)$

LEVEL 3

Matrix Matching Type (M.M.T.):

SET-I

Column-I

34. $\log(a+b) + \log(a-b) - \log(a^2-b^2) =$

35. $\log 2 + 2\log 5 - \log 3 - 2\log 7 =$

36. $\log_7 8 \times \log_8 7 =$

37. $\frac{1}{2}\log 9 + \frac{1}{4}\log 81 + 2\log 6 - \log 12 =$

Column-II

(A) $\log\frac{50}{147}$

(B) log 27

(C) 0

(D) 1

(E) 3 log 3

SET-II

Column - I

38. $\frac{1}{2+\sqrt{3}}$ [R.F of Dr]

39. $64^{1/2}$

40. $16^{3/4}$

41. $7^{1/2}\ 8^{1/2}$

Column - II

(A) Irrational

(B) $\sqrt{56}$

(C) 8

(D) $2-\sqrt{3}$

Assertion Reason Type (A.R.T.):

(A) Both assertion and reason are true and reason is the correct explanation of assertion

(B) Both assertion and reason are true but reason is not the correct explanation of assertion

(C) Assertion is true but reason is false

(D) Assertion is false but reason is true

42. Assertion: Every integer is a rational number.

Reason: Every integer 'm' can be expressed in the form $\frac{m}{1}$.

43. Assertion: $2+\sqrt{6}$ is an irrational number.

Reason: Sum of a rational number and an irrational number is always an irrational number.

44. Assertion: If $x = 5 - 2\sqrt{6}$, then $x + \frac{1}{x}$ is irrational.

Reason: If $x = 4 + \sqrt{15}$ then $\left(x + \frac{1}{x}\right)^3$ is 512.

Statement Type (S.T.):

(A) Both statements are correct

(B) Both statements are incorrect

(C) Statement I is correct statement II is incorrect

(D) Statement I is incorrect statement II is correct

45. Statement-I: $2\sqrt[3]{40} + 3\sqrt[3]{625} - 4\sqrt[3]{320} = 3\sqrt[3]{5}$.

Statement-II: Like surds can be added and subtracted.

46. Statement-I: $\left[\left(\sqrt{3}+5\right)\left(\sqrt{3}-5\right)\right]$ is a rational number.

Statement-II: $\left(\sqrt{3}+5\right)\left(\sqrt{3}-5\right)$ = –22, which is a rational number.

47. Statement-I: The rationalizing factor of $\frac{1}{a+b\sqrt{x}}$ is $a - b\sqrt{x}$.

Statement-II: The order of the surd $\sqrt[5]{7}$ is 7.

Integer Type Questions (I.T.Q.):

48. $\frac{\left(\sqrt{32}+\sqrt{48}\right)}{\left(\sqrt{8}+\sqrt{12}\right)}$ = ____.

49. If x = $\frac{1}{\left(2-\sqrt{3}\right)}$, then value of $(x^3–2x^2–7x+5)$ = ____.

50. The number of integers between $-\sqrt{8}$ and $\sqrt{32}$ is ____.

51. If $2^{x-1}+2^{x+1}$ = 320 then the value of x is ____.